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An unified potential model of quark-antiquark interaction and meson spectroscopy

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Abstract : Assuming the quark-antiquark potential to contain a vector part consisting of a series of terms and a confining part having a Lorentz vector as well as a Lorentz scalar parts, we give the phenomenological results in a quark potential model obtained from a non-relativistic form of hamiltonian using Bethe-Salpeter equation within the framework of quantum chromodynamics We find that such a model should always give non-zero values for $\overline{M}_1 - M'_1$ where $\overline{M}_1$ = C.O.G of ^3P_J states and M'_1 is the mass of the $^1P_1$ state. The results match well with the observed spectroscopy of S-wave and P-wave mesons.

Keywords : QCD, potential model, meson spectra.

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1. Introduction

Although there have been lot of studies on the long distance part of the quark-antiquark interaction, the short distance vector part has been considered to be the colour-Coulomb interaction [1]. It will be interesting to see the effect on the phenomenological aspect of meson spectroscopy if we consider a general potential model having a series of terms in the vector part of the quark-antiquark interaction in a potential model motivated by quantum chromodynamics. This model is termed as an 'unified potential model' because most of the existing models can be shown to follow from this model under special conditions. We have given analytical expressions for the masses of S-state and P-state mesons, and we arrive at some general conclusions based on the mass formulas. The purpose of this work is to give a detailed phenomenological discussion of meson spectroscopy and fix the parameters relevent for the description of spectra. Several new as well as interesting conclusions emerge from this analysis.

The paper is organised as follows. In §2 we give introduction to the unified potential model. In §3 we describe the model and apply it to S-states of meson and give the method of calculation of parameters. In §4 we give the phenomenology of P-states. In §5 we give results, discussions and conclusions.

2. The unified potential model

We now briefly introduce the unified potential model. Following Lichtenberg [2], the vector-part due to multiple gluon exchange-like effect is written as

$$V = r^b \sum_{n=0}^{\infty} b_n r^n, \; b \geq -1 \; b_0 < 0. \tag{2.1}$$

The scalar part is written as [3]

$$S = ar^{\beta}. \tag{2.2}$$

In scalar-vector model of confinement [3], the confining potential contains an effective scalar part

$$S_{\text{eff}} = (1-\eta)S = (1-\eta)ar^{\beta}, \; a > 0, \; \beta > 0 \tag{2.3}$$

and an effective vector part

$$V_{\text{eff}} = V + \eta S = r^b \sum_{n=0}^{\infty} b_n r^n + \eta a r^{\beta}. \tag{2.4}$$

We restrict b to take the values -1, 0, 1 so that eq. (2.1) gives

$$V = b_0 r^{-1} + b_1 + b_2 r, \qquad (b = -1); \tag{2.5}$$

$$V = b_0 + b_1 r + b_2 r^2, \qquad (b = 0); \tag{2.6}$$

$$V = b_0 r + b_1 r^2 + b_2 r^3, \qquad (b = 1). \tag{2.7}$$

Now, we cannot work with infinite terms as otherwise we have a large number of parameters. So we have to truncate the series after some terms. Now following the general properties of the static potential [4] given by gauge theory, the conditions to be satisfied are

$$\frac{dv_0}{dr} > 0, \tag{2.8}$$

$$\frac{d^2 v_0}{dr^2} \leq 0. \tag{2.9}$$

For $V_0 = r^b \sum_{n=0}^{\infty} b_n r^n + \eta a r^{\beta}$, the above inequalities take the forms (2.10)

$$\sum_{n=0}^{\infty} b_n (b+n) r^{b+n-1} + a\eta\beta r^{\beta-1} > 0, \tag{2.11}$$

$$\sum_{n=0}^{\infty} b_n (b+n)(b+n-1) r^{b+n-2} + a\eta\beta(\beta-1) r^{\beta-1} \leq 0. \tag{2.12}$$

For $b = -1$, 0 or 1 and $\beta = 1$ the above inequalities suggest $b_0 < 0$.

3. The model Hamiltonian

The full Hamiltonian is

$$H = H_0 + H' \tag{3.1}$$

where
$$H_0 = \sum m_i + \sum \frac{P_i^2}{2m_i} + V_0, \tag{3.2}$$

$$V_0 = V(r) + S(r). \tag{3.3}$$

The vector potential $V(r)$ is given by

$$V(r) = r^b \sum_{n=0}^{\infty} b_n r^n \tag{3.4}$$

and the confining part which is a scalar-vector admixture is given by

$$S(r) = \eta a r^{\beta} + (1-\eta) a r^{\beta}. \tag{3.5}$$

The interaction Hamiltonian which is treated as a perturbation is given by

$$H' = H_{\mathrm{kin}} + V_{\mathrm{spin}}, \tag{3.6}$$

where
$$H_{\mathrm{kin}} = -\sum \frac{(p_i)^4}{(2m_i)^3} = f_0(1/m_i^3 + 1/m_j^3) \tag{3.7}$$

and
$$V_{\mathrm{spin}} = \frac{1}{m_i m_j}\Big[\lambda_1 F_1(r)\left\{\frac{(m_i+m_j)^2 + 2m_i m_j}{4m_i m_j} L.S - \frac{m_i^2 - m_j^2}{4m_i m_j} L.(S_i - S_j)\right\}$$

$$+\lambda_2 F_2(r) T_{ij} + \lambda_3 F_3(r) S_i . S_j \Big]. \tag{3.8}$$

For $L = 0$ state, V_{spin} reduces to

$$V_{\mathrm{spin}} = \frac{\lambda_3}{m_i m_j}\left[F_3(r) + 4\pi b_0 \delta(\boldsymbol{r})\right]. \tag{3.9}$$

The mass formula with non-zero anomalous magnetic moment [3], can be written as

$$M(^{2s+1}L_J) = \sum m_i + M_L + \frac{1}{m_i m_j}\Bigg[\lambda_1 F_1(r)\left\{\frac{(m_i+m_j)^2 + 2m_i m_j}{4m_i m_j} L.S. \right.$$

$$\left. - \frac{m_i^2 - m_j^2}{4m_i m_j}\left\langle L.(S_i - S_j)\right\rangle\right\} + \lambda_2 \left\langle F_2(r)\right\rangle \left\langle T_{ij}\right\rangle + \lambda_3 \left\langle F_3(r)\right\rangle \left\langle S_i . S_j\right\rangle\Bigg]$$

$$+ f_0\left(1/m_i^3 + 1/m_j^3\right), \tag{3.10}$$

where m_i ($i = 1,2$) is the mass of the quark and $f_0 = -\frac{1}{8}\left\langle P_i^4 \right\rangle$

$$M_L = \langle V_0 \rangle. \tag{3.11}$$

The tensor operator T_{ij} is given by

$$T_{ij} = -4S_i.S_j + 12(S_i.\hat{r})(S_j.\hat{r}). \tag{3.12}$$

Here $\lambda_1 = 3/2$, $\lambda_2 = 1/12$. $\lambda_3 = 2/3$ are constants. $F_1(r)$, $F_2(r)$, $F_3(r)$ are the radial functions which are defined as

$$F_1(r) = \frac{2V'_{\rm eff}}{r} - \frac{S'_{\rm eff}}{r}, \tag{3.13}$$

$$F_2(r) = -V''_{\rm eff} + \frac{V'_{\rm eff}}{r}, \tag{3.14}$$

$$F_3(r) = \nabla^2 V_{\rm eff}(r). \tag{3.15}$$

3.1. S-states of quarkonia :

The mass of the S-state is given by

$$M = m_i + m_j + M_0 + \frac{\lambda_3}{m_i m_j}\left[\langle F_3(r)\rangle + 4\pi b_0 \langle \delta(r)\rangle\right]\langle s_i.s_j\rangle$$
$$+ f_0(1/m_i^3 + 1/m_j^3). \tag{3.16}$$

The mass of a vector meson is given by

$$M_v = m_i + m_j + M_0 + \frac{1}{6m_i m_j}\left[\langle F_3(r)\rangle + 4\pi b_0 \langle \delta(r)\rangle\right]$$
$$+ f_0(1/m_i^3 + 1/m_j^3) \tag{3.17}$$

and the mass of a psudo-scalar meson is given by

$$M_p = m_i + m_j + M_0 - \frac{1}{2m_i m_j}\left[\langle F_3(r)\rangle + 4\pi b_0 \langle \delta(r)\rangle\right]$$
$$+ f_0(1/m_i^3 + 1/m_j^3). \tag{3.18}$$

Hence, the mass difference between the vector and pseudo-scalar mesons is given by

$$M_v - M_p = \frac{2}{3m_i m_j}\left[\langle \nabla^2 V_{\rm eff}(r)\rangle + 4\pi b_0 \langle \delta(r)\rangle\right]. \tag{3.19}$$

For heavy-light systems we have

$$M_v + M_p \approx 2(m_i + m_j).$$

Hence, $$M_v^2 - M_p^2 = \delta M_{ij}^2 = (M_v + M_p)(M_v - M_p)$$

$$\frac{4}{3\mu_{ij}}\left[\langle \nabla^2 V_{\rm eff}(r)\rangle + 4\pi b_0 \langle \delta(r)\rangle\right], \tag{3.20}$$

where μ_{ij} is the reduced mass of the meson containing i-th quark and j-th anti-quark or vice-versa. Here i and j are the quark flavour indices, $\{i,j \in (u,d,s,c,b,t\}$. Now using

$$r = x\mu^{-1/(2+\beta)} \tag{3.21}$$

as in [2], we have

$$\langle\delta(r)\rangle = \langle\delta(x)\rangle\mu^{3/(2+\beta)} \tag{3.22}$$

and

$$\langle\nabla^2 V_{\text{eff}}\rangle = \sum_{n=0}^{\infty} b_n(b+n)(b+n+1)\langle x^{b+n-2}\rangle\mu^{-(b+n-2)/(2+\beta)}$$
$$+a\eta\beta(\beta+1)\langle x^{\beta-2}\rangle\mu^{(2-\beta)/(2+\beta)}. \tag{3.23}$$

Hence, the difference of masses of a vector and pseudo-scalar mesons is

$$M_v - M_p \equiv \delta_{ij} = \sum_{n=0}^{\infty} b_n(b+n)(b+n+1)\langle x^{b+n-2}\rangle\mu^{-(b+n-2)/(2+\beta)}$$
$$+a\eta\beta(\beta+1)\langle x^{\beta-2}\rangle\mu^{(2-\beta)/(2+\beta)} + 4\pi b_0\langle\delta(x)\rangle\mu^{3/(2+\beta)}. \tag{3.24}$$

$$M_v^2 - M_p^2 \equiv \delta M_{ij}^2 = \frac{4}{3\mu_{ij}}\sum_{n=0}^{\infty} b_n(b+n)(b+n+1)\langle x^{b+n-2}\rangle\mu^{-(b+\beta+n)/(2+\beta)}$$
$$+a\eta\beta(\beta+1)\langle x^{\beta-2}\rangle\mu^{(2-\beta)/(2+\beta)} + 4\pi b_0\langle\delta(x)\rangle\mu^{(1-\beta)/(2+\beta)}. \tag{3.25}$$

Martin [5] gave an empirical relation for the mesons containing one heavy and one light quark, which suggests that the difference of squared masses of vector and pseudo-scalar mesons *i.e.* $(M_v^2 - M_p^2)$ is flavour-independent (≈ 0.56 GeV^2). We use this flavour-independence of δM_{ij}^2 to find the relation among the potential parameters. From the relation

$$\frac{\partial}{\partial\mu}(\delta M_{ij}^2) = 0, \tag{3.26}$$

we get

$$\sum_{n=0}^{\infty} -b_n(b+n)(b+n+1)\langle x^{b+n-2}\rangle\left(\frac{b+\beta+n}{2+\beta}\right)\mu^{-(b+2\beta+n+2)/(2+\beta)}$$
$$+ a\eta\beta(\beta+1)\left(\frac{2-\beta}{2+\beta}\right)\langle x^{\beta-2}\rangle\mu^{-(2+3\beta)/(2+\beta)}$$
$$+ 4\pi b_0\langle\delta(x)\rangle\left(\frac{1-\beta}{2+\beta}\right)\mu^{-(1+2\beta)/(2+\beta)}. \tag{3.27}$$

Now for $\beta = 1$, the above condition reduces to

$$\sum_{n=0}^{\infty} b_n(b+n)(b+n+1)^2\langle x^{b+n-2}\rangle\mu^{-(b+4+n)/3} = -4a\eta\langle x^{-1}\rangle\mu^{-5/3}. \tag{3.28}$$

Now for different values of b (= 0, 1, −1), L.H.S of eq. (3.28) is expanded and comparing with terms containing equal powers of μ, we get values of the parameters of the vector part of the potential in terms of 'a' and 'η'. These values are given in the Table 1. The predicted values of M_v, M_p, $M_v - M_p$ are given in Tables 2 and 3 respectively.

Table 1. The values of the potential parameters for S-states of mesons.

		b_0	$?_1$		
1	$b = 0$	$b_0 = 0$	$b_1 = -a\eta$	$b_n = 0$ for $n \geq 0$	$V = b_0 - a\eta r$
1	$b = -1$	$b_0 = 0$	$b_1 = 0$ $b_2 = -a\eta$,	0, for $n \geq 3$	$V = \frac{b_0}{r} + b_1 - a\eta r$
1	$b = 1$	$b_0 = -a\eta$		$b_n = 0$, for $n \geq 1$	$V = -a\eta r$

Table 2. The predicted values of M_v and M_p are compared with the experimental data for the quark masses $m_u = 0.333$, $m_d = .333$, $m_s = .500$, $m_c = 1.6$, $m_b = 5$, $m_t = 180$ GeVs.

Quark content	M_v		M_p	
	(This Work)	Expt. [7]	(This Work)	Expt [7]
qq	0.76629	0 7673	0.19288	0 1380
qs	0 88995	0 8931	0 49195	0.4950
qc	2 04476	2.0089	1 90553	1 8678
qb	5 87085	5 3248	5.82382	5 2788
qt	203.70438	?	203.70305	?
sc	2.60654	2 1100	2.49881	1 9685
sb	7 28636	?	7 24852	?
st	247 74831	?	247 7472	?

Table 3. Values of $M_v - M_p$ in GeV for the quark masses $m_u = 0.333$, $m_d = 333$, $m_s = 500$, m_c 1.6, $m_b = 5$, $m_t = 180$ GeVs. (! are the input values)

Quark content	This work	$M_v - M_p$				
		SC & DG [8]	Lucha *et al* [6]			Expt [7]
			NR	SR	Salpeter	
qq	0 57341	0.5942	–	0 023	0.6300	0.6300
qs	0 39800 !	0.3981				0 3981
qc	0.13923	0 1411				0.1411
qb	0 04703	0.0526				0.0460
qt	0.00135	0 0016				
sc	0 10773	0.1258				
sb	0.03784	0 0505				
st	0.00111	0.0016				
cc	0.05592	0 1181 !	0.099	0.006	0.140	0.1181
bb	0.01510	0.0054				

4. The P-states of self-conjugate mesons ($c\bar{c}, b\bar{b}$)

For $L = 1$ state, the mass formulas for self-conjugate mesons reduce to the following forms :

$$M(^3P_0) = M_0 = M + 2f_0/m_i^3 - 2\lambda_1 A_{11} - 4\lambda_2 A_{12} + \lambda_3 A_{13}/4, \tag{4.1}$$

$$M(^3P_1) = M_1 = M + 2f_0/m_i^3 - \lambda_1 A_{11} + 2\lambda_2 A_{12} + \lambda_3 A_{13}/4, \tag{4.2}$$

$$M(^3P_2) = M_2 = M + 2f_0/m_i^3 + \lambda_1 A_{11} - \tfrac{2}{5}\lambda_2 A_{12} + \lambda_3 A_{13}/4, \tag{4.3}$$

$$M(^1P_1) = M_1' = M + 2f_0/m_i^3 - \tfrac{3}{4}\lambda_3 A_{13}, \tag{4.4}$$

where M is the common mass given by the spin-independent part of the potential and the radial functions A_{1i} are given by

$$A_{1i} = \frac{1}{m_q^2}\langle F_i(r)\rangle, \quad i = 1,2,3. \tag{4.5}$$

Using eq. (3.13–3.15), the expressions for the radial functions are

$$F_1(r) = \sum_{n=0}^{\infty} b_n(b+n)r^{b+n-2} + a\eta\beta r^{\beta-2} - \frac{1}{3}a\beta r^{\beta-2}, \tag{4.6}$$

$$F_2(r) = \sum_{n=0}^{\infty} b_n(b+n)(2-b-n)r^{b+n-2} + a\eta\beta(2-\beta)r^{\beta-2}, \tag{4.7}$$

$$F_3(r) = \sum_{n=0}^{\infty} b_n(b+n)(b+n+1)r^{b+n+2} + a\eta\beta(\beta+1)r^{\beta-2}. \tag{4.8}$$

The centre of gravity of the ^3P_J states is

$$\overline{M}_1 = \frac{1}{9}\left[5M_2 + 3M_1 + M_0\right] = M + 2f_0/m_i^3 + 6A_{13}. \tag{4.9}$$

Eqs. (4.4) and (4.5) give

$$\overline{M}_1 - M_1' = \frac{2}{3}A_{13}. \tag{4.10}$$

The confining part of the spin-orbit interaction is given by

$$M_{SO}^{CF} \equiv -\frac{1}{m_i^2}\left\langle \frac{S'_{\text{eff}}}{r}\right\rangle = -(1-\eta)a\beta\left\langle r^{\beta-2}\right\rangle. \tag{4.11}$$

And the quantity (R) which measures the structure of the spin-orbit force is

$$R = \frac{M_2 - M_1}{M_1 - M_0} = \frac{2}{5}\,\frac{\sum_{n=0}^{\infty}(13+b+n)A - 5B - 18\eta\beta + \eta\beta b}{\sum_{n=0}^{\infty}(5-b-n)A - B + 6\eta\beta - \eta\beta B} \tag{4.12}$$

where
$$A = b_n(b+n)\langle r^{b+n-2}\rangle, \tag{4.13}$$
$$B = a\beta\langle r^{\beta-2}\rangle. \tag{4.14}$$

For $\beta = 1, b = 0, b_0 = 0, b_1 = -a\eta; b_n = 0$ for $n \geq 2$ so that

$$V = b_0 + b_1 r + b_2 r^2$$

and
$$R = \frac{2}{5}\frac{14b_1\langle 1/r\rangle + 30b_2 - 5a\langle 1/r\rangle - 17\eta a\langle 1/r\rangle}{4b_1\langle 1/r\rangle + 6b_2 - a\langle 1/r\rangle + 5\eta a\langle 1/r\rangle}. \tag{4.15}$$

For $\beta = 1, b = 1, b_0 = -a\eta, b_n = 0$ for $n \geq 1$, so that $V = b_0 r + b_1 r^2$ and

$$R = \frac{2}{5}\frac{14b_0\langle 1/r\rangle + 30b_1 - 5a\langle 1/r\rangle - 17\eta a\langle 1/r\rangle}{4b_0\langle 1/r\rangle + 6b_1 - a\langle 1/r\rangle + 5\eta a\langle 1/r\rangle}. \tag{4.16}$$

For $\beta = 1, b = -1, b_0 = 0, b_1 = 0, b_2 = -a\eta, b_n = 0$ for $n \geq 3$,

$$R = \frac{2}{5}\frac{-12b_0\langle 1/r^{-3}\rangle + 14b_2\langle 1/r\rangle - 5a\langle 1/r\rangle - 17a\eta\langle 1/r\rangle}{-6b_0\langle 1/r^{-3}\rangle + 4b_2\langle 1/r\rangle - a\langle 1/r\rangle + 5a\eta\langle 1/r\rangle}. \tag{4.17}$$

For $\beta = 1$

$$M_{SO}^{CF} = -\frac{(1-\eta)}{2m_i^2}a\langle 1/r\rangle. \tag{4.18}$$

Now using the scaling given in eq. (3.21) we get

$$M_{SO}^{CF} = -\frac{(1-\eta)}{2m_i^2}a\langle x^{-1}\rangle\mu^{1/3}, \tag{4.19}$$

$$\left(M_{SO}^{CF}\right)_{b\bar{b}} = \left(M_{SO}^{CF}\right)_{c\bar{c}}\left(m_c/m_b\right)^{5/3}. \tag{4.20}$$

For $\beta = 2$, the expression of M_{SO}^{CF} reduces to

$$M_{SO}^{CF} = a(\eta - 1)m_q^{-2}, \tag{4.21}$$

$$\left(M_{SO}^{CF}\right)_{b\bar{b}} = \left(M_{SO}^{CF}\right)_{c\bar{c}}\left(m_c/m_b\right)^2. \tag{4.22}$$

For $b = -1, \beta = 1, V = b_0/r + b_1 + b_2 r$,

$$\overline{M}_1 - M_1' = \frac{2}{3m_i^2}\left[2b_2\langle 1/r\rangle + 6b_3 + 2a\eta\langle 1/r\rangle\right]. \tag{4.23}$$

For $b = 0, \beta = 1$,

$$\overline{M}_1 - M_1' = \frac{2}{3m_i^2}\left[2b_1\langle 1/r\rangle + 6b_2 + 2a\eta\langle 1/r\rangle\right]. \tag{4.24}$$

For $b = 1$, $\beta = 1$,

$$\overline{M}_1 - M_1' = \frac{2}{3m_i^2}\left[2b_0\langle 1/r\rangle + 6b_1 + 2a\eta\langle 1/r\rangle\right]. \tag{4.25}$$

Again using the scaling given in eq. (3.21) and taking only up to linear terms in the vector potential (eqs. (2.5–2.7)), both the eqs. (4.23) and (4.24) give

$$\left(\overline{M}_1 - M_1'\right)_{c\bar{c}} = \left(\overline{M}_1 - M_1'\right)_{b\bar{b}}\left(m_b / m_c\right)^{5/3}. \tag{4.26}$$

The results of the studies on R, M_{SO}^{CF} and $\overline{M}_1 - M_1'$ are given in Table 4.

Table 4. Values of R, $\overline{M}_1 - M_1'$ and M_{SO}^{CF} in MeV for $b\bar{b}$ and $c\bar{c}$ with $m_c = 1.6$ and $m_b = 5.0$ GeVs. (! are the input values).

β	b	R		M_{SO}^{CF}		$\overline{M}_1 - M_1'$	
		$c\bar{c}$	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$
		(0.48)	(0.664)	(–25.9)	(–7 7)	(0.00)	(5.4 ± 1.5)
1	0 1	0.664	0.664!	–25.9!	–3.88	36 07	5 4!
	–1	$R_{c\bar{c}}$ =	$R_{b\bar{b}}$	–	–	–	
2	–			–25.9!	–2.65		

5. Results, discussion and conclusion

The various parameters related to a general vector interaction (b_0, b_1, b_2 *etc*) and the confining parameters (a,β) are given in Table 1. We consider here, three types of vector potentials. Among these, the potentials $V = b_0 - a\eta r$ and $V = -a\eta r$ are not interesting, but the combination of linear and colour-Coulomb term is very interesting. The linear part $-a\eta r$ comes from short distance vector part and if $\eta < 1$ the resultant effect of 'ar' and '$-a\eta r$' will effectively give confinement. From the Table 2, we find that the over all agreement of the mass spectrum with the experiment is good. The results on $M_v - M_p$ are given in the Table 3 and comparison is made with previous works of Lucha *et al* [6] and experimental data [7]. In the heavy-light sector, agreement with the experiment is very good. In the light-light sector, deviation from the experiment is about 57 MeV. This model gives low value of $M_v - M_p$ compared to the model of ref. [8]. From the Table 3, it is also clear that the model cannot reproduce the $M_v - M_p$ for self-conjugate states, as expected. The deviation from the experiment is more prominent in the heavy-heavy sector. This is due to the fact that potential parameters given in the Table 1 are meant for heavy-light mesons, for which $M_v^2 - M_p^2$ is assumed to be constant. Actually there is a slight variation of $M_v^2 - M_p^2$ and this may be a probable cause of slight deviation of our results from experimental values. Lucha *et al* [6] have given a comparative study of non-relativistic, semi-relativistic and relativistic (Salpeter) description of quarkonia for different potentials. They have reproduced a few light and heavy mesons by minimising the quantity

$$x^2 = \sum\left[(M_i - M_i^{\exp}) / \nabla M_i\right]^2$$

where M_i and $M_i^{\exp}$ are the theoretical and experimental masses of a meson respectively and ΔM_i is the error. In this process only experimentally observed states can be studied. We compare our result with that for Coulomb plus linear potential [6].

For the P state, instead of calculating individual masses, we have calculated $(\overline{M}_1 - M_1')$, R and M_{SO}^{CF} (Table 4). From eqs. (4.23) to (4.25) we find that $\overline{M}_1 - M_1'$ is non-zero for any value of η as obtained from experiment. If the vector part of the static potential contains terms only up to linear in 'r', then from eq. (4.26), we get $(\overline{M}_1 - M_1')_{c\bar{c}} = 30.07$ MeV with $(\overline{M}_1 - M_1')_{b\bar{b}} = 5.4$ MeV as input. For $b = 0$ and 1 neglecting terms greater than linear, we get $R_{c\bar{c}} = R_{b\bar{b}}$ (eqs. 4.15 and 4.16). For $b = -1$, $R_{c\bar{c}} = R_{b\bar{b}}$ (eq. 4.17), but numerical values cannot be calculated as number of parameters are greater than the number of states. As η is less than unity, M_{SO}^{CF} is always negative. In the limit of $\eta = 1$, M_{SO}^{CF} becomes zero for both linear and harmonic confinement. Using eqs. (4.20) and (4.22) and taking experimental value of $(M_{SO}^{CF})_{c\bar{c}} = -25.9$ MeV as the input, we get $(M_{SO}^{CF})_{b\bar{b}} = -3.88$ MeV for linear confinement and -2.65 MeV for harmonic confinement.

Thus, we find that a general short distance vector part gives a contribution to the potential model, and thereby introduces new features in the phenomenological description of a quarkonia.

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